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USING BOX-JENKINS MODEL TO FORECAST FISHERY DYNAMICS I:

IDENTIFICATION, ESTIMATION, AND CHECKING

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In this two part paper, the use of Box-Jenkins models for modeling and forecasting fisheries is explored. Part I explores in some detail the process of identifying and estimating a Box-Jenkins model, and how forecasts are made and updated. A model developed to forecast skipjack tuna, Katsuwonus pelamis, catches in Hawaii is used to illustrate the methodology. In part II, the strengths and weaknesses of models developed to forecast yellowfin tuna, Thunnus albacares, catches in the eastern Pacific and salmon runs in the Skeena River are discussed.

Forecasting techniques have improved greatly in the last few years, and research in fisheries management generally has not kept up with these advances. There are good reasons for considering forecasting models. Firstly, the needs of good forecasts are different from those of the usual stock assessment techniques. These latter techniques primarily are aimed at determining the (equilibrium) status or health of the stock. While the fit to the observed data used to estimate the stock assessment models vary from case to case, these models on the whole do not successfully predict the actual history of the stock for the next few time periods of interest.

A second reason for considering forecasting models is the weak statistical basis underlying both the general production model (Fox 1970, 1971, 1975) and such models as the Ricker spawner-recruit curve (Ricker 1954). Each of these techniques scales the dependent variable (catch or recruits) by the independent variable (effort or spawners), and then performs a least squares fit against a function of the independent variable. Several authors (Chayes 1949; Eberhardt 1970;

Atchley et al. 1976) have demonstrated that this procedure can introduce severe bias into the estimation procedure, so that even two uncorrelated random variables would appear to have a significant relationship. The general production model often uses a function not of effort, but of a weighted sum of the effort over the past several years. After scaling, effort is then estimated as a lagged variable of itself. Johnston (1972) shows that using ordinary least squares procedures in this instance again biases the estimated "goodness of fit."

Most importantly, however, is the fact that most fishery data are highly autocorrelated through time: An examination of the residuals in Fox (1971, Fig. 3B) clearly exhibit autoregressive behavior. The same can be said for most residuals from spawner-recruit curves. Granger and Newbold (1977) and Newbold and Davies (1978) have termed using ordinary least squares in these instances "spurious regression" and demonstrate how the estimated fit of the model is biased upward when the errors are misspecified.

Box-Jenkins models and other forecasting techniques are usually specifically designed to estimate parameters when the data are autocorrelated, including seasonal data. Moreover, the models are stochastic, rather than deterministic, reflecting the variability observed in most fisheries. Our preference for Box-Jenkins models over other forecasting techniques is more practical than anything else. Box-Jenkins models are well documented (see for example Anderson 1975; Box and Jenkins 1976; Granger and Newbold 1977), they are empirically constructed from the data rather than force fitted, and there are several packages available to perform the necessary computations. The results presented here were calculated using a package originally developed by David Pack at Ohio State University; and now available through Automated Forecasting Systems.

II. THE DATA AND THE UNDERLYING MODEL

The data to be analyzed are landings of skipjack tuna by approximately 12 boats on Oahu 1964 through 1978. Each boat rarely stays out more than a day or two, and the raw data consist of the daily landings, broken down by boat, and for four size classes, large, medium, small, and extra small. For purposes of analysis, the data have been aggregated into monthly totals, with the total number of fishing trips used as the measure of fishing effort. The monthly catch and effort for the period

Figs.1,2 1964-78 are plotted in Figures 1 and 2.

There are several causes for the observed seasonal variability. Firstly, the tuna are only available seasonally in large numbers. Secondly, price considerations, particularly around Christmas and New Years when there is large demand, tend to spur fishing even when availability is low. Thirdly, with only 12 boats fishing, if 1 or 2 boats are not able to fish for a few weeks, the catch will drop sharply. Finally, environmental factors, particularly weather (such as bad seas) will affect the landings since the boats are unable to fish.

Folklore in Hawaii has it that the catch remains pretty much the same each year, no matter how many boats fish. Comitini (1977) examined the fishery using dummy variables and ordinary least squares to estimate a Cobbs-Douglas production fuction. He concludes among other things that natural fluctuations in resource availability are significant, but does not include them in his analysis, nor does he provide a means for forecasting future catch. The National Marine Fisheries Service, using a regression model based on the previous year's catch, water temperature, and salinity at the start of the year make yearly predictions that have been mixed in accuracy. Monthly forecasts have been thought to be impractible or impossible due to the variability in the data.

A feature of the data not examined in this paper is that prior to 1973, the catch of the large tuna and the total catch "track" together closely. After 1973 this is not so. There are several possible explanations for this, including the mercury scare in the early 1970's for large tuna, increased foreign fishing around Hawaii, and increased catches elsewhere in the Pacific. While this change in size composition does not affect our modeling of the total catch (as will be seen), it is certainly a trend worth analyzing in greater detail if more accurate forecasts are to be obtained.

Box-Jenkins models are based on autoregressive-integrated-moving-average models, or ARIMA models. These are linear, stochastic models that can describe fairly complex behavior. Unlike Parrish and MacCall (1978) say, who have gone to more highly nonlinear equations to model the fluctuations in fishery data, the approach here is to use less complex, linear models.

The modeling is based on the properties of stationary time series. A time series x_t is stationary if it has mean zero, and if the covariance between events x_t, x_{t-s} depends only on s and not on t. Many series are stationary after removing the estimated (observed) mean. Others have to be differenced in order to achieve stationary. It is convenient to use the backshift operator $B^j x_t = x_{t-j}$ when describing lags or differencing. The initial step then is to transform and difference the data as necessary to achieve stationarity. Given the new series $z_t = (1 - B^d)x_t$, a mixture of autoregressive and moving average models are sought. Autoregressive models are lags on the past history of the time series:

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \ldots + \phi_p z_{t-p} + a_t$$

or equivalently:

$$(1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p) z_t = a_t$$

and moving average models are lags on the past noise or error a_t:

$$z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q}$$

or:

$$z_t = (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q) a_t$$

A model that has both moving average and autoregressive parameters is a mixed autoregressive moving average model. The general representation is:

$$(1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)(1 - B^d) x_t = (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q) a_t.$$

III. MODEL IDENTIFICATION

The first step in the modeling process is to use properties of the data to tentatively identify a model. In Box-Jenkins techniques, even if a multivariate model (i.e., a model based on catch and effort) is the ultimate goal, univariate models of each series are first constructed. Often the univariate model produces forecasts that are almost as accurate as the multivariate model forecast.

Our procedure was to fit initially a model from the data for January 1964 through July 1977, and then forecast to December 1978. If the forecasts seemed reasonable, the parameters would be reestimated using the data for January 1964 through December 1978, and then monthly forecasts for 1979 would be made. (To make clear the feedback nature of identification, estimation, and checking in Box-Jenkins models, results from models fixed to 163 mo and 180 mo of data are intermingled, but clearly labeled.) In both cases, the forecasts are of data points not used in identifying the model or estimating the parameters.

A tentative model can be identified by estimating the regular and partial autocorrelation functions for each series. These are shown in Figures 3-4. Significant is the undamped sinusoidal behavior of each, with a period of 12 mo. Failure of both the regular and partial autocorrelation functions to go to zero is a sign of a nonstationary series, and the need for differencing. In this instance, a seasonal model is suggested, so that twelfth differences were taken, that is $z_t = (1 - B^{12})x_t$.

Figs. 3-4

The estimated regular and partial autocorrelation functions for the differenced catch and effort series are given in Tables 1 and 2. There are several ways to proceed from here, but suspecting a multiplicative seasonal model, appendix 9.1 in Box and Jenkins (1976) was consulted. This lists special characteristics of the autocovariances of the multiplicative seasonal models most often encountered. In particular, a seasonal model with period s of the form:

$$z_t = (1 - \theta_1 B - \theta_2 B^2)\ (1 - \theta_1 B^s - \theta_2 B^{2s}) a_t \qquad (3.1)$$

has the special characteristics:

$$p_{s-2} = p_{s+2}$$

$$p_{s-1} = p_{s+1}$$

$$p_{2s-2} = p_{2s+2}$$

$$p_{2s-1} = p_{2s+1}$$

and should have a significant autocorrelation at lag s. For s = 12,
Tables 1,2 Tables 1 and 2 both show a highly significant autocorrelation at lag 12. Moreover, the implied equalities for effort are: -0.08 = -0.08; -0.20 = -0.18; 0.05 = -0.07; 0.04 = 0.00. The implied equalities for catch are: -0.12 = -0.15; -0.21 = =0.21; -0.08 = -0.06; -0.08 = -0.06. As these estimated values come close to the theoretical properties of model (3.1), this was accepted as a tentative model for the two time-series.

IV. ESTIMATION AND CHECKING

There are two methods generally used to estimate parameters for Box-Jenkins parameters. The first involves maximizing the conditional likelihood, the second the unconditional. In some packages, the program will request if it is desired to suppress backforecasting. Backforecasting is used in the unconditional estimation. When backforecasting is used, the algorithm is more sensitive to starting values and takes longer to execute. The estimates, however, usually have a smaller residual sum of squares than when backforecasting is suppressed. A procedure often used is to first estimate the parameters suppressing backforecasting, and then use these estimates as starting values for the unconditional estimation.

The estimate for both catch and effort for model (3.1) are given in

Tables 3,4 Tables 3 and 4. There are two major techniques for checking for model inadequacy. The first is to calculate the estimated regular and partial autocorrelations of the model residuals, and the second is to overfit the model. The estimated regular and partial autocorrelations for the residuals

Tables 5,6 from each series when backforecasting is used are given in Tables 5 and 6. For the effort series, there is no sign of lack of fit. For the catch series, however, additional terms of lag for three and four are suggested. An overspecified model:

$$z_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3 - \theta_4 B^4)\ (1 - \Theta_1 B^{12})$$

Tables 7,8 was fit to both time series. The results are summarized in Tables 7 and 8. The regular and partial autocorrelation functions (not shown) show no sign of additional lags or trend. Box and Jenkins (1976, p. 291) give a portmanteau chi square statistic to test whether the residual series can adequately be described as white noise. The test statistic, given in Tables 7 and 8, suggest no reason to question the models' adequacy.

V. TRANSFER FUNCTION MODELS

If both the catch time series, say y_t, and the effort time series, say x_t, have been suitably transformed so that the resulting series are stationary, a transfer function can be estimated of the form:

$$(1-\delta_1 B-\delta_2 B^2 - \ldots -\delta_r B^r)x_t = (\omega_0-\omega_1 B-\omega_2 B^2 -\ldots-\omega_s B^s)y_{t-b} + \eta_t$$

where η_t is not assumed to be white noise, but itself can be modeled as an autoregressive-moving average process of a_t.

The procedure for identifying and estimating a transfer function model are similar to those for the univariate model, except that attention is focused on the estimated cross-correlation function between the "prewhitened" catch and effort series. Series are prewhitened if they are reduced to the residuals left from a given model. In this instance, both series are prewhitened by the univariate model for effort estimated in section IV. The estimated correlation function, impulse response function, and residual

Table 9 noise autocorrelation function are given in Table 9. The estimated autocorrelation function for the noise is similar to the original univariate autocorrelations, suggesting a noise model of the form:

$$\eta_t = (1-\theta_1 B-\theta_2 B^2-\theta_3 B^3-\theta_4 B^4)\ (1-\Theta_1 B^{12})a_t \tag{5.1}$$

Based on guidelines in Box and Jenkins (1977, p. 386-388) and knowledge of the fishery, two models were hypothesized:

$$(1-B^{12})y_t = (\omega_0)\ (1-B^{12})x_t + \eta_t \qquad (5.2)$$

and:

$$(1-\delta_1 B-\delta_2 B^2)(1-B^{12})y_t = (\omega_0-\omega_1 B-\omega_2 B^2)(1-B^{12})\,x_t+\eta_t \qquad (5.3)$$

Tables 10,11 Tables 10 and 11 summarize the estimates when backforecasting is used in estimating the parameters for (5.2) and (5.3). The chi square statistics show no reason to suspect model inadequacy. The residuals show no significant cross-correlation with total catch, when $^1/\sqrt{180}$ (180 observations in the series) is used as a rough standard error. The residual autocorrelation function shows spikes around lag 15 that are higher than would be desired, but overall the fit is reasonable, and the model residuals could reasonably be modeled as white noise.

VI. DISCUSSION AND FORECASTS

The three models that were fit to forecast the total monthly skipjack tuna catch can be better interpreted when written in difference equation form. The univariate model becomes:

$$y_t = y_{t-12} + (a_t + 0.538\,a_{t-1} + 0.438\,a_{t-2} + 0.412\,a_{t-3} + 0.309\,a_{t-4})$$
$$- (0.996\,a_{t-12} + 0.535\,a_{t-13} + 0.436\,a_{t-14} + 0.410\,a_{t-15} + 0.308\,a_{t-16}) \qquad (6.1)$$

That is, the catch this month is equal to the catch the same month last year, plus the difference between a weighted sum of this year's forecasting errors compared to last year's. The model checks to see how far off the mean trend this year has been compared to last year, and adjusts the predicted catch according. The model that has been derived is completely consistent with the local folklore, only it gives a method to adjust the forecast to include the year to year variability. The other expected result is that the residual mean square is large (115, 170), despite the fact that the residuals have been reduced to white noise. The model was constructed from the data, but is consistent with the experience and perceived knowledge of people familiar with the fishery.

This impression of a yearly cycle with variability is reinforced when examining the polynomial representation of the model (3.1). The value of Θ is nearly one. Thus, the term $(1 - B^{12})$ appears on both sides of the equation, and can be cancelled. Abraham and Box (1978) show that this is sufficient reason to suspect a deterministic cosine function trend.

This approach is not used since the deterministic trend would not self-correct to the past errors in the forecasts.

The two multivariate models are:

$$\begin{aligned} y_t = {} & 8.003\,x_t + (y_{t-12} - 8.003\,x_t) \\ & + (a_t + 0.489\,a_{t-1} + 0.326\,a_{t-2} + 0.149\,a_{t-3} + 0.175\,a_{t-4}) \qquad (6.2) \\ & - (0.996\,a_{t-12} + 0.487\,a_{t-13} + 0.325\,a_{t-14} + 0.148\,a_{t-15} + 0.174\,a_{t-16}) \end{aligned}$$

and

$$y_t = 8.186\,x_t + (0.866\,y_{t-1} - 6.742\,x_{t-1}) - (0.708\,y_{t-2} - 7.313\,x_{t-3})$$

$$+ (y_{t-12} - 8.186\,x_{t-12}) + (0.866\,y_{t-13} - 6.752\,x_{t-13}) - (0.708\,y_{t-14} - 7.313\,x_t$$

$$+ (a_t + 0.470\,a_{t-1} + 0.332\,a_{t-2} + 0.172\,a_{t-3} + 0.217\,a_{t-4}) \qquad (6.3)$$

$$- (0.995\,a_{t-12} + 0.468\,a_{t-13} + 0.331\,a_{t-14} + 0.171\,a_{t-15} + 0.216\,a_{t-16})$$

The model (6.12) predicts that catch this month is a function of the relative catch and effort in the same month last year, corrected by the relative forecasting errors the last 4 mo this year less the same errors the year before. The model (6.3) is similar, except that the catch and effort for the 4 mo previous, both this year and last, are used in the prediction.

Forecasts can be made using any of the three models by substituting expected values for unknown values. If T is the base period of the forecast, and the last observed period, then for $\ell > 0$, the expectation of $a_{t+\ell}$ is zero. The expected effort series can be generated by the univariate effort model, and for each period $T + \ell$, the predicted catch and effort are used in the forecast as the expected value.

Table 12

Monthly forecasts for 1979 are given in Table 12. It can be seen that the models perform adequately. January is the worst month, but January 1979 had unusually severe weather and relatively little fishing. The forecasts rise a little too quickly for April, but by May the forecasts are fairly accurate.

More importantly, a desirable feature of any forecast is an ability to constant self-correct to the observed forecasting errors. Table 13 gives the same three forecasts after the January-April 1979 data has been included in the data set. The updated forecasts lower the May estimate, since January-February were bad months, but increase the predicted catch in late summer. Experience with each of the models is that while there may be considerable month to month error, the forecasts quickly self-correct. Over a 12-15 mo period, the forecasting errors can be expected to cancel each other out, providing a relatively accurate forecast for the year. The updated forecasts predict a yearly catch of about 3,300 metric tons (MT), with the three forecasts differing less than 100 MT. This is almost 1,000 MT better than 1978. The earlier catch record, plus observations from the fishermen, would seem to support this optimistic forecast.

Table 13

Improved forecasts might be expected if effective trips per month is used as the input series instead of total trips per month. Also, breaking the catch into size classes may improve the forecast, since the dynamics of the size classes differ, and the total catch dynamics is a mixing of each of these series. These possibilities will be reported on in future papers.

VII. SUMMARY

Box-Jenkins models have been proposed as an alternate model for forecasting fishery data. ARIMA models provide maximum likelihood estimators that are not biased when the data is seasonal and autocorrelated, and when a viable is lagged on itself. Techniques are explored which allow the model to be constructed from the data up, rather than from questionable theoretical models. The procedure is illustrated on skipjack tuna catches in Hawaii, which traditionally has been considered too variable to forecast on a monthly basis in a reasonable manner.

Part II of this paper will examine models of the yellowfin tuna catch in the Pacific and salmon runs on the Skeena River. In part II, less emphasis will be given to how to identify the model, and more emphasis will be given to the problems of forecasting complex systems with relatively short time series. This leads to large estimated standard errors for parameters with values much different from zero. Alternate methods of estimating the transfer function between catch and effort will also be examined.

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LITERATURE CITED

ANDERSON, O. D.

1975. Time series analysis and forecasting. The Box-Jenkins approach. Butterworths, Lond./Boston,

ATCHLEY, W. R., C. T. GASKINS, AND D. ANDERSON.

1976. Statistical properties of ratios: Empirical results. Syst. Zool. 25:137-148.

BOX, G. E. P., AND G. M. JENKINS.

1976. Time series analysis: Forecasting and control. (Rev. ed.), Holden-Day, San Francisco, 575 p.

CHAYES, F.

1949. On ratio correlation in petregraphy. J. Geol. 57:239-254.

COMITINI, S.

1977. An economic analysis of the state of the Hawaiian skipjack tuna fishery. Sea Grant Tech. Rep. UNIHI-SEAGRANT-TR-78-01, 46 p.

EBERHARDT, L. L.

1970. Correlation, regression and density dependence. Ecology 51: 306-310.

FOX, W. W. JR.

1970. An exponential surplus-yield model for optimizing exploited fish populations. Trans. Am. Fish. Soc. 99:80-88.

1971. Random variability and parameter estimation for the generalized production model. Fish. Bull., U.S. 69:659-580.

1975. Fitting the generalized stock production model by least-squares and equilibrium approximation. Fish. Bull., U.S. 73:23-37.

GRANGER, C. W. J., and P. NEWBOLD.

1977. Forecasting economic time series. Acad. Press, N.Y., 333 p.

JOHNSTON, J.

1972. Econometric methods, 2nd ed. McGraw-Hill, N.Y., 437 p.

NEWBOLD, P., and N. DAVIES.

1978. Error mis-specification and spurious regressions. Econometrica

PARRISH, R. H., and A. D. MacCALL.

1978. Climatic variation and exploitation in the Pacific mackerel fishery. Calif. Dep. Fish Game, Fish Bull. 167, 110 p.

RICKER, W. E.

1954. Stock and recruitment. J. Fish. Res. Board Can. 11:559-623.

Table 1.--Autocorrelation functions for 12th differenced effort series.

Lag	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Regular auto.	0.39	0.17	0.20	0.16	0.10	0.04	0.03	0.07	-0.02	-0.08	-0.20	-0.45	-0.18	-0.08
S.E.	0.08	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.12	0.12
Partial auto.	0.39	0.03	0.15	0.03	0.01	-0.04	0.00	0.00	-0.08	-0.07	-0.19	-0.39	0.15	0.04

Lag	15	16	17	18	19	20	21	22	23	24	25	26	27
Regular auto.	-0.18	-0.13	-0.12	-0.17	-0.15	-0.17	-0.12	0.05	0.04	0.02	0.00	-0.07	0.03
S.E.	0.12	0.12	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.13	0.13	0.13	0.13
Partial auto.	-0.03	0.02	-0.07	-0.14	-0.01	-0.02	-0.05	0.16	-0.08	-0.19	0.05	-0.12	0.03

Table 2.--Autocorrelation functions for 12th differenced catch.

Lag	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Regular auto.	0.58	0.40	0.33	0.20	0.11	0.05	0.01	0.02	-0.06	-0.12	-0.21	-0.38	-0.21	-0.15
S.E.	0.08	0.11	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.13	0.13	0.14	0.14
Partial auto.	0.58	0.09	0.10	-0.07	-0.03	-0.04	-0.00	0.04	-0.11	-0.07	-0.17	-0.29	0.28	0.03

Lag	15	16	17	18	19	20	21	22	23	24	25	26	27
Regular auto.	-0.16	-0.12	-0.08	-0.09	-0.08	-0.12	-0.10	-0.08	-0.08	-0.09	-0.06	-0.06	-0.05
S.E.	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
Partial auto.	-0.01	-0.06	-0.02	-0.07	0.02	-0.05	-0.04	-0.04	-0.11	-0.17	-0.19	-0.01	-0.02

Table 3.--Parameter estimates for effort model

$$z_t = (1 - \theta_1 B - \theta_2 B^2)\ (1 - \Theta_1 B^{12} - \Theta_2 B^{24})\ a_t\ .$$

Parameter	Estimate suppressing backforecasting	Standard error	Estimate using backforecasting	Standard error
θ_1	-0.38349	0.07942	-0.44756	0.07886
θ_2	-0.11326	0.07996	-0.12795	0.07911
Θ_1	0.5894	0.08122	0.99493	0.00650
Θ_2	0.00069	0.08609	--	--
χ^2 statistic on residuals	26.894 with 44 d.f.		37.319 with 45 d.f.	
Residual mean square	1,018.60		755.270	
Residual standard error	31.915		27.482	
Residual mean	1.629		0.5338	

Based on 180 observations.

Table 4.--Parameter estimates for catch model

$$z_t = (1 - \theta_1 B - \theta_2 B^2)(1 - \Theta_1 B^{12} - \Theta_2 B^{24}) a_t .$$

Parameter	Estimate suppressing backforecasting	Standard error
θ_1	-0. 54100	0.08190
θ_2	-0.22745	0.08235
Θ_1	0.75314	0.08718
Θ_2	0.05184	0.09256
χ^2 statistic on residuals	27.470 with 43 d.f.	
Residual mean square	165410	
Residual standard error	406.71	
Residual mean	17.506	

Based on 163 observations.

Table 5.--Estimated autocorrelation function for residuals of effort model.

Lag	1	2	3	4	5	6	7	8	9	10	11	12
Auto.	0.01	0.03	0.09	0.06	0.04	-0.11	-0.05	0.04	-0.09	0.02	0.03	-0.01
S.E.	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Lag	13	14	15	16	17	18	19	20	21	22	23	24
Auto.	0.05	-0.01	-0.04	-0.09	-0.08	-0.09	-0.07	-0.08	-0.16	0.10	0.07	-0.02
S.E.	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08

Table 6.--Estimated autocorrelation function for residuals of catch model.

Lag	1	2	3	4	5	6	7	8	9	10	11	12
Auto.	0.04	0.11	0.23	0.06	0.06	-0.05	0.00	0.10	-0.03	0.04	0.01	0.00
S.E.	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09

Lag	13	14	15	16	17	18	19	20	21	22	23	24
Auto.	0.05	-0.01	-0.07	-0.01	-0.00	-0.04	0.00	-0.01	-0.06	0.01	-0.04	-0.03
S.E.	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09

Table 7.--Parameter estimates for effort model

$$(1-B^{12})\,x_t = (1-\theta_1 B-\theta_2 B^2-\theta_3 B^3-\theta_4 B^4-\theta_5 B^5)\,(1-\Theta_1 B^{12})\,a_t\,.$$

Parameter	Estimate suppressing backforecasting	Standard error	Estimate using backforecasting	Standard error
θ_1	-0.36746	0.08004	-0.43862	0.07930
θ_2	-0.14976	0.08412	-0.18144	0.08590
θ_3	-0.16111	0.08458	-0.15377	0.08617
θ_4	-0.17096	0.08454	-0.16298	0.08593
θ_5	-0.11547	0.08089	-0.17291	0.07998
Θ_1	0.59065	0.06431	0.99483	0.00033
χ^2 statistic on residuals	20.696 with 42 d.f.		27.494 with 42 d.f.	
Residual mean square	1,000.40		752.67	
Residual standard error	31.629		27.435	
Residual mean	0.82151		0.35175	

Based on 180 observations.

Table 8.--Parameter estimates for the catch model

$$(1 - B^{12})\, x_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3 - \theta_4 B^4 - \theta_5 B^5)\,(1 - \Theta_1 B^{12})\, a_t\,.$$

Parameter	Estimate suppressing backforecasting	Standard error	Estimate using backforecasting	Standard error
θ_1	-0.55368	0.07972	-0.53771	0.07462
θ_2	-0.35882	0.08989	-0.43825	0.07543
θ_3	-0.33817	0.09056	-0.41197	0.01144
θ_4	-0.24282	0.09012	-0.30909	0.07479
θ_5	-0.12294	0.07994	-0.14974	0.07440
Θ_1	0.76951	0.05062	0.99585	0.00825
χ^2 statistic on residuals	15.092 with 42 d.f.		20.384 with 42 d.f.	
Residual mean square	143240		115170	
Residual standard error	378.47		339.37	
Residual mean	2.1150		3.3299	

Based on 180 observations.

Table 9.--Estimated cross-correlation function, impulse response function, and noise autocovariance function for a catch-effort transfer model.

Lag	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Est. cross-corr.	0.651	0.080	0.070	0.086	-0.033	0.044	-0.098	0.099	0.103	-0.017	0.043	-0.040	-0.20	0.026
Est. noise auto.	--	0.49	0.21	0.16	0.16	0.10	0.07	0.03	0.13	0.14	-0.05	-0.14	-0.26	-0.05
S.E.		0.10	0.12	0.12	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.14
Est. impulse response wts.	8.409	1.035	0.903	1.111	-0.431	0.566	-1.269	1.276	1.334	-0.215	0.556	-0.517	-1.555	0.338

Lag	14	15	16	17	18	19	20	21	22	23	24	25	26
Est. cross-corr.	-0.109	0.003	-0.098	0.014	-0.110	-0.037	0.006	-0.006	-0.108	0.012	-0.108	-0.001	-0.108
Est. noise auto.	0.05	-0.12	-0.16	-0.01	-0.05	-0.12	-0.12	-0.16	-0.11	-0.18	-0.21	-0.09	-0.08
S.E.	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.15	0.15	0.15	0.15
Est. impulse response wts.	-1.404	-0.038	-0.415	0.043	-1.271	0.185	-1.422	-0.475	0.080	-0.075	-1.393	0.181	-1.390

Table 10.--Parameter estimates for transfer model

$$(1-B^{12})y_t = \omega_0(1-B^{12})x_t + (1-\theta_1 B-\theta_2 B^2-\theta_3 B^3-\theta_4 B^4)\ (1-\Theta_1 B^{12})\,a_t\,.$$

Parameter	Estimate suppressing backforecasting	Standard error	Estimate using backforecasting	Standard error
ω_0	7.5989	0.69403	8.0003	0.83561
θ_1	-0.47621	0.07993	-0.48894	0.07851
θ_2	-0.32874	0.08734	-0.32633	0.08541
θ_3	-0.17034	0.08803	-0.14853	0.08666
θ_4	-0.20033	0.07905	-0.17506	0.07822
Θ_1	0.83384	0.05271	0.99587	0.00707
χ^2 statistic on residuals	34.953 with 43 d.f.		32.018 on 43 d.f.	
Residual mean square	83,323		71,300	
Residual standard error	288.66		267.02	
Residual mean	-15.152		0.18650	

Based on 180 observations

Table 11.--Parameter estimates for transfer model

$$(1 - \delta_1 B - \delta_2 B^2)\ (1 - B^{12})x_t = (\omega_0 - \omega_1 B - \omega_2 B^2)\ (1 - B^{12})x_t$$
$$+ (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3 - \theta_4 B^4)\ (1 - \Theta_1 B^{12})a_t.$$

Parameter	Estimate suppressing backforecasting	Standard error	Estimate using backforecasting	Standard error
δ_1	0.01286	0.30389	0.86672	0.22308
δ_2	0.88121	0.28641	-0.70763	0.21659
ω_0	7.3488	0.73352	8.1855	0.82832
ω_1	-1.3011	2.16847	6.7421	1.71214
ω_2	6.8509	2.34577	-7.3133	1.58459
θ_1	-0.49924	0.08302	-0.46980	0.08013
θ_2	-0.29495	0.09102	-0.33234	0.08870
θ_3	-0.16384	0.09191	-0.17199	0.09012
θ_4	-0.13639	0.08352	-0.21746	0.08098
Θ_1	0.83311	0.05511	0.99543	0.00623
χ^2 statistic on residual	33.067 with 43 d.f.		38.906 with 43 d.f.	
Residual mean square	85,673		69,066	
Residual standard error	292.70		262.80	
Residual mean	-1.9979		-2.4666	

Based on 180 observations

Table 12.--Catch forecasts for 1979 from the three models (MT).

	Model			
Month	$(1-B^{12})y_t = \omega_0(1-B^{12})x_t + \eta_t$	$(1-\delta_1 B-\delta_2 B^2)\ (1-B^{12})y_t = (\omega_0-\omega_1 B - \omega_2 B^2)(1-B^{12})x_t + \eta_t$	Univariate	Observed
Jan.	102.24	157.48	159.97	52.6488
Feb.	78.91	123.32	117.81	74.1184
Mar.	121.86	118.83	108.40	102.4088
Apr.	202.05	169.75	175.82	131.0658
May	423.40	406.87	423.95	470.5450
June	595.39	605.68	598.17	
July	666.16	684.99	607.07	
Aug.	528.09	535.73	523.14	
Sept.	297.96	294.92	291.97	
Oct.	224.28	216.64	222.96	
Nov.	173.99	168.83	172.94	
Dec.	133.22	131.61	132.58	
Total	3,544.55	3,614.65	3,594.78	

Table 13.--Updated forecasts of total catch (MT).

	Model			
Month	$(1 - B^{12})y_t = \omega_0(1 - B^{12})x_t + \eta_t$	$(1 - \delta_1 B - \delta_2 B^2)(1 - B^{12})y_t = (\omega_0 - \omega_1 B - \omega_2 B^2)(1 - B^{12})x_t + \eta_t$	Univariate	Observed
May	393.214	382.430	401.874	470.545
June	547.014	586.400	589.524	
July	644.638	705.137	668.895	
Aug.	500.151	527.945	521.456	
Sept.	293.130	283.067	289.516	
Oct.	220.557	197.953	222.806	
Nov.	174.567	164.720	173.594	
Dec.	130.947	136.831	133.148	
Total	2,904.218	2,983.983	3,000.813	

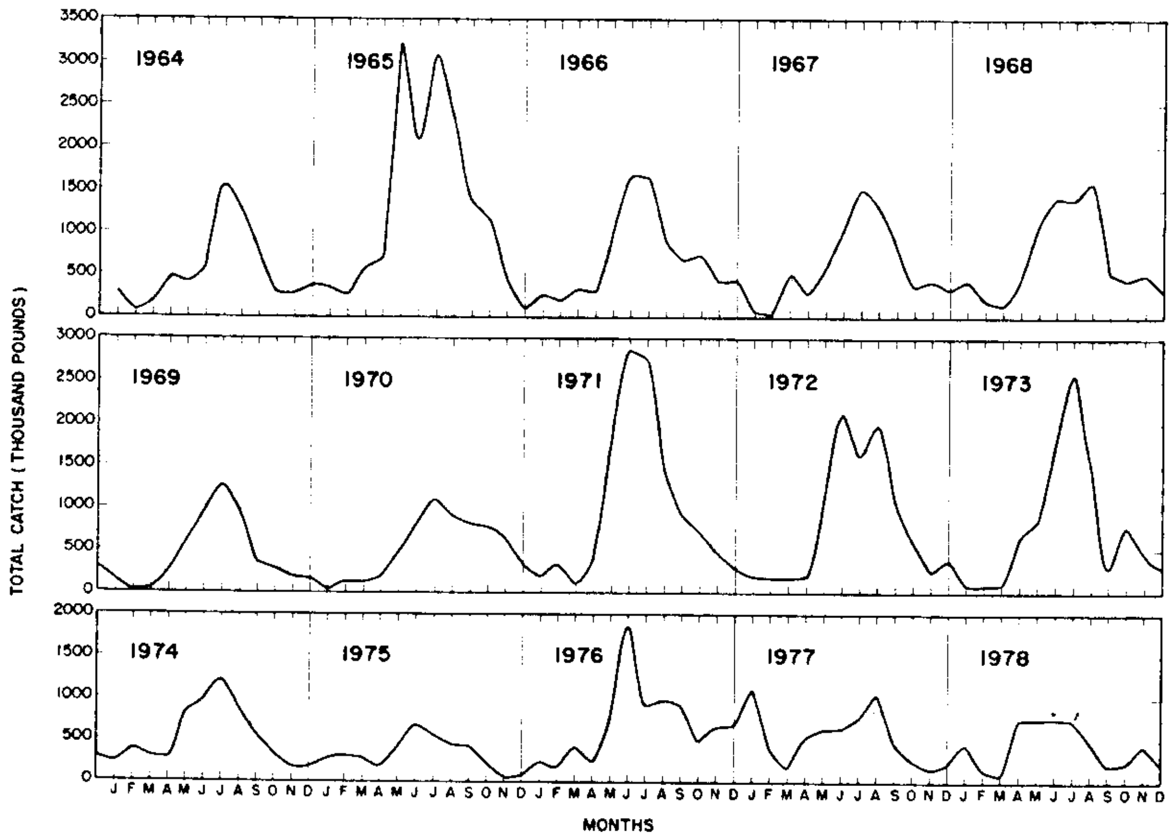

Figure 1.--Level of skipjack tuna catch by month, 1964-78.

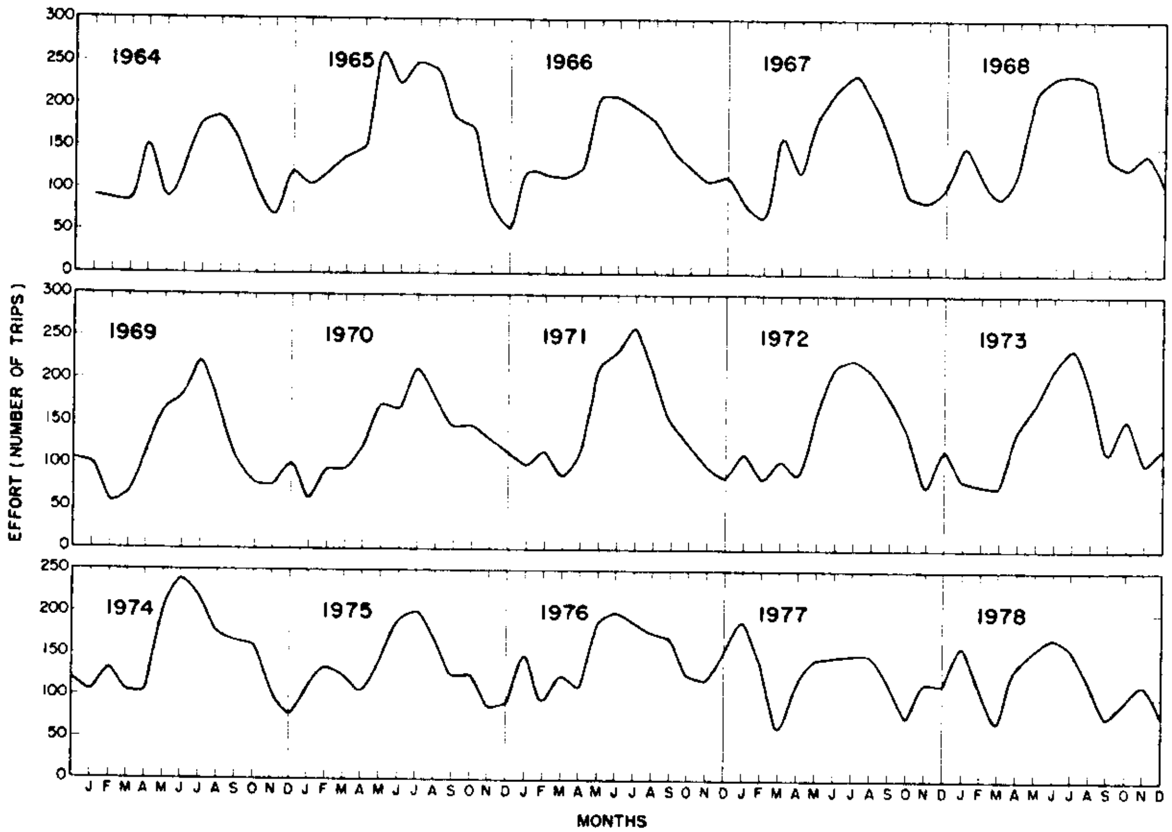

Figure 2.--Number of fishing trips per month, 1964-78.

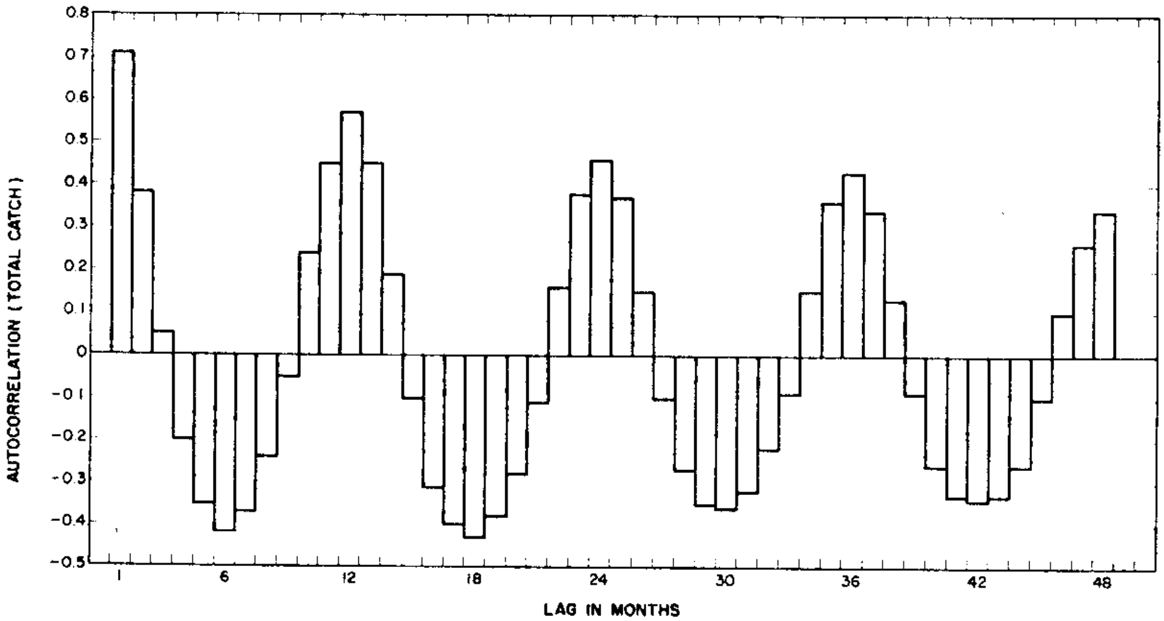

Figure 3.--Estimated total catch autocorrelation function.

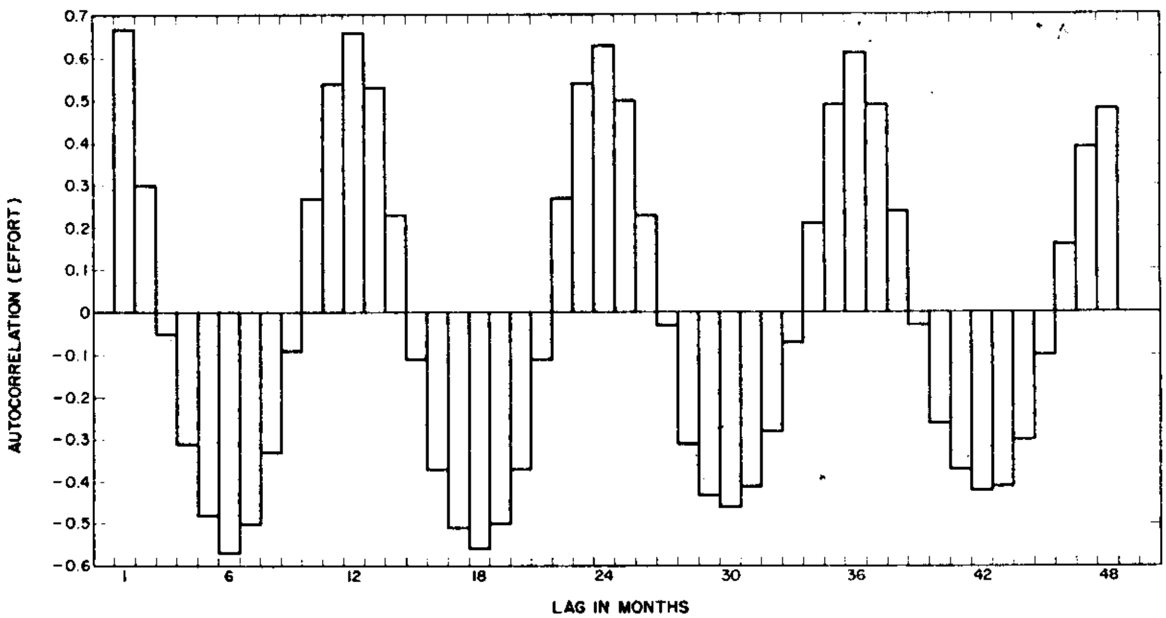

Figure 4.--Estimated effort autocorrelation function.